Modelos Estáticos de Equilibrio General en Mathematica

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Resumen: El presente trabajo tiene por objetivo brindar una exposición de los Medelos Estaticos de Equilibrio General. Para llevar a cabo dicha tarea se presenta una revisión teórica de tal tópico complementada con aplicaciones en Matemática 4.1 . Se brindan además rutinas de programación en Matemática que automatizan la tarea de resolución de dichos problemas. De esta manera, se logra cumplir el fin de fomentar el uso de tal método cuantitativo minimizando el esfuerzo de aprendizaje y resolución.

Palabras clave: Modelo Clásico, Modelo Keynesiano, Equilibrio General

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Para llevar a cabo la exposición de tales modelos se brindan a continuación los supuestos en los que se basan:

1. la tasa de depreciación del capital permanece constante a lo largo del análisis, es decir, $\partial \delta = 0$.

2. K es fijo (tanto a nivel empresa individual como la economía en su agregado).

3. Existen n empresas idénticas que producen un único producto (el mismo tanto para el consumo como para la inversión en bienes de capital). Asimismo poseen la misma función de producción:

   $$Y = F(K, N) \qquad \textit{donde} \qquad Y = \sum_{i=1}^{n} Y_i \;,\quad N = \sum_{i=1}^{n} N_i \;,\quad K = \sum_{i=1}^{n} K_i$$

   La tecnología de producción es cóncava, es decir:

   $$F_N > 0 \;,\; F_K > 0 \;,\; F_{NN} < 0 \;,\; F_{KK} < 0 \;,\; F_{NK} > 0$$

4. Mercado del factor (de la producción) trabajo, N, competitivo.

La notación a emplear se detalla como apéndice al final[3].

[3] Apéndice I, página.

Modelo sin fricciones: caso Clásico.

Queda así un sistema formado por 7 ecuaciones que incluyen 7 variables endógenas, a saber:

Variables endógenas: Y, N, P, C, I, r, w/P

Variables exógenas: K, G, T, M, B, δ, π

Las ecuaciones de dicho sistema son:

- La función de producción agregada de la economía es:

  (1) $Y = F(K, N)$

- La demanda de trabajo de las empresas surge del proceso de maximización del beneficio:

  (2) $w/P = F_N(N, K)$

- La condición clásica de equilibrio en el mercado laboral al nivel de pleno empleo (asegurado por ajustes flexibles de la tasa salarial) esta dada por:

  (3) $N = N(w/P)$ $\quad$ *donde* $N^S = N(w/P)$ $\quad \partial N^S / \partial \, w/P > 0$

- La función demanda de consumo[4] es:

  (4) $C = C\left(Y - T - \delta K - \frac{M+B}{P}\pi ; r - \pi \right)$ $\quad \partial C / \partial Y_d > 0$, $\partial C / \partial (r-\pi) < 0$

  donde $Y_d = Y - T - \delta K - \frac{M+B}{P}\pi$

- La función demanda de inversión (del tipo keynesiano)[5] es:

[4] Función adaptada para la presente revisión. La función original empleada por Sargent es

$$C = C\left(Y - T - \delta K - \frac{M+B}{P}\pi + (q(K, N, r-\pi, \delta) - 1); r - \pi \right)$$

[5] Función adaptada para la presente revisión. La función original empleada por Sargent es:

$$\frac{\partial K}{\partial t} \equiv I \equiv I\left(\frac{F_K - (r - \pi + \delta)}{r - \pi} \right), \; I' > 0 \text{ o como } I \equiv I(q-1)$$

donde q es la definición debida a Tobin: $q = \frac{F_K - (r - \pi + \delta)}{r - \pi} + 1 \equiv q(K, N, r - \pi, \delta)$

(5) $\dfrac{\partial K}{\partial t} \equiv I \equiv I\left(F_K - (r - \pi + \delta)\right) \quad , \quad {\partial I}\Big/{\partial (q-1)} = I' > 0$

en donde

$$I_1 = {\partial I}\Big/{\partial K} < 0 \ \ , \ I_2 = {\partial I}\Big/{\partial N} > 0 \ \ , \ I_3 = {\partial I}\Big/{\partial (r-\pi)} < 0 \ \ , \ I_4 = {\partial I}\Big/{\partial \delta} < 0$$

- El ingreso de la economía en términos agregados se define como:
  (6) $Y = C + I + G + \delta K$

- Por último, la condición de equilibrio del mercado monetario es
  (7) $M\big/P = m(r, Y)$

La diferencial total del sistema es (con $\partial M + \partial B = 0$):

(1´) dy == Fk dk + Fn dN

(2´) dwp Š Fnk dk + Fnn dN

(3´) dN Š Ns dwp

(4´) dM- M dp p Š p [illegible] r dr + my dy

(5´) dy Š dc + dIn + dg + d dk

(6´) dIn Š I [illegible] dk [illegible] I2 dN + I3 [illegible]

(7´) dc Š C1 [illegible] y- [illegible] T- d dk- [illegible] pi [illegible] B [illegible] B [illegible] 2 [illegible] pi [illegible] + B [illegible] r- [illegible] pi

Modelo con fricciones: Caso Keynesiano.

La condición clásica de equilibrio en el mercado laboral al nivel de pleno empleo (asegurado por ajustes flexibles de la tasa salarial) no se cumple en este caso, debido a que la tasa nominal de salario es rígida no permitiendo que las fuerzas del mercado consigan el equilibrio.

Queda así un sistema formado por 6 ecuaciones que incluyen 6 variables endógenas a saber:

Variables endógenas: $\mathrm{Y}, \mathrm{N}, \mathrm{P}, \mathrm{C}, \mathrm{I}, \mathrm{r}$

Variables exógenas: $\mathrm{K}, \mathrm{G}, \mathrm{T}, \mathrm{M}, \mathrm{B}, \delta, \pi, w$

Las ecuaciones de dicho sistema son:

- La función de producción agregada de la economía es:
  (1) $Y = F(K, N)$

- La demanda de trabajo de las empresas surge del proceso de maximización del beneficio:
  (2) $w/P = F_N(N, K)$

- La función demanda de consumo[6] es:
  (3) $C = C\left(Y - T - \delta K - \frac{M+B}{P}\pi ; r - \pi\right) \quad \partial C/\partial Y_d > 0 \;,\; \partial C/\partial (r-\pi) < 0$

  $$\textit{donde} \quad Y_d = Y - T - \delta K - \frac{M+B}{P}\pi$$

- La función demanda de inversión (del tipo keynesiano)[7] es:
  (4) $\frac{\partial K}{\partial t} \equiv I \equiv I\left(F_K - (r - \pi + \delta)\right) \;,\; \partial I/\partial (q-1) = I' > 0$

  en donde

  $$I_1 = \partial I/\partial K < 0 \;,\; I_2 = \partial I/\partial N > 0 \;,\; I_3 = \partial I/\partial (r-\pi) < 0 \;,\; I_4 = \partial I/\partial \delta < 0$$

- El ingreso de la economía en términos agregados se define como:
  (5) $Y = C + I + G + \delta K$

- Por último, la condición de equilibrio del mercado monetario es
  (6) $M/P = m(r, Y)$

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[6] Idem pie de página N° 5

[7] Idem pie de página N° 6

La diferencial total del sistema es (con $\partial M + \partial B = 0$):

(1´) dy = [illegible] Fk d[illegible] + Fn dN

(2´) [illegible]dw p[illegible] w dp [illegible] 2 Š [illegible]Fnk dk + [illegible]nn dN

(3´) dM- Mdp pŠ p [illegible]r d[illegible] + my dy

(4´) dy Š dc+ dIn + dg + d dk

(5´) dIn Š I[illegible] dk [illegible] I2 dN + I3 [illegible]

(6´) dc Š C1 [illegible]y- [illegible] - d dk- [illegible]i [illegible] + [illegible]B [illegible] + B [illegible] 2 [illegible] + B [illegible]r- [illegible]pi

EJERCICIO 7. Explique y profundice sobre el título 2 del Cap. 2 de Sargent.

ESTABILIDAD

Además de ajustarse al alza el precio (y la tasa de interés) ante un exceso de demanda agregada (ante un exceso de demanda de saldos reales) al nivel inicial en consideración, el modelo keynesiano (a diferencia del clásico) permite variación del producto ofrecido al variar el precio.

Por ello en el siguiente análisis se supondrá que durante el ajuste al equilibrio, el sistema permanezca siempre sobre la curva de oferta agregada, así la ecuación de su pendiente

$$\left.\frac{\partial P}{\partial Y}\right|_{OA} = \frac{F_{NN}\,P}{F_N^{\,2}} \qquad o \qquad \left.\frac{\partial Y}{\partial P}\right|_{OA} = -\frac{F_{NN}\,w}{F_N^{\,2}\,P^2}$$

se verifica continuamente en el proceso.

Se postula como ecuación de movimiento para p y r las siguientes ecuaciones diferenciales:

$$\frac{\partial P}{\partial t} = \alpha\left[C\left(Y - T - \delta K\,; r - \pi\right) + I\left(F_K - (r - \pi + \delta)\right) + G + \delta K - Y\right],\ \alpha' > 0\ ,\ \alpha(0) = 0$$

$$\frac{\partial r}{\partial t} = \beta\left[m(r, Y) - M/P\right],\ \beta' > 0\ ,\ \beta(0) = 0$$

Estas ecuaciones describen la presión hacia el ajuste, aunque tal ajuste ocurre en forma instantánea (el modelo determina todas las variables endógenas a cada momento). Por lo tanto, se considera que estas ecuaciones diferenciales se cumplen cuando las velocidades de ajuste α´ y β´ tienden a infinito.

Vemos que la condición de estabilidad solo requiere que la pendiente de la curva LM sea mayor a la de curva IS, no restringiendo el análisis únicamente a casos en que la IS tenga su pendiente normal.

EJERCICIO 8. Explique y profundice sobre el título 3 del Cap. 2 de Sargent.

ANÁLISIS DE LAS INFLACIONES DE "DE EMPUJÓN DE LOS COSTES" Y " DE TIRÓN DE LA DEMANDA"

El resultado obtenido en el presente modelo para la curva de **oferta agregada** es[8]:

$$dy \text{ ® } dk\, Fk + \frac{Fn^2 \left(\frac{dp}{p} + \frac{dw}{w}\right)}{Fnn}$$

donde se observa la dependencia del producto ofrecido en la economía de la tasa de variación de los precios y el salario nominal (en conjunto la tasa de variación del salario real), las condiciones tecnológicas y el flujo de nuevo capital.

El producto ofrecido varía en forma directa con el nivel de precios, así ($\left.\partial Y \middle/ \partial P\right|_{OA} =$)

$$-\frac{Fn^2}{Fnn\, p} > 0$$

La **demanda agregada** es (con $\partial K = 0$):

$$dp \text{ ® } -\frac{mr\, p^2 \left([illegible] pi + \frac{-dg\, Fn + C1\, (dT - [illegible]) [illegible] dy\, (Fn - I2 [illegible])}{Fn\, (C2 + I3 [illegible])} + \frac{dM + dy\, my\, p^2}{mr\, p^2} \right)}{M}$$

donde se observa la dependencia del producto demandado en la economía de la variación de los precios, oferta de dinero, el nivel de precios las condiciones tecnológicas y parámetros funcionales.

El producto demandado varía en forma inversa con el nivel de precios, así ($\left.\partial P \middle/ \partial Y\right|_{DA} =$)

$$\frac{[illegible] 1 + C1 [illegible] r + I2\, mr - [illegible] F [illegible] C2 + I3 [illegible] p^2}{Fn\, [illegible] C2 + I3 [illegible] M [illegible]} < 0$$

Es interesante destacar los efectos que producen los cambios en la oferta de sus determinantes, pues estos producen lo que se conoce como **inflación del empujón de costos**.

Así, respectivamente, se obtiene:

- ($\left.\partial P \middle/ \partial w\right|_{OA} =$) y ($\left.\partial^2 P \middle/ \partial w^2\right|_{OA} =$) son: $\frac{p}{w}$, $-\frac{p}{w^2}$

- ($\left.\partial Y \middle/ \partial w\right|_{OA} =$) y ($\left.\partial Y^2 \middle/ \partial w^2\right|_{OA} =$)son:

[8] Suponiendo $M + B = 0$ y $\partial M + \partial B = 0$

$\frac{Fn^2}{Fnn\ w}$ $\qquad$ $-\frac{Fn^2}{Fnn\ w^2}$

$\left(\partial P / \partial F_N \Big|_{OA} = \right)$ y $\left(\partial^2 P / \partial F_N{}^2 \Big|_{OA} = \right)$ son:

$\frac{2\,(dy - dk\,Fk\,Fnn\,p)}{Fn^3}$ $\qquad$ $-\frac{6\,(dy - dk\,Fk\,Fnn\,p)}{Fn^4}$

- $\left(\partial Y / \partial F_N \Big|_{OA} = \right)$ y $\left(\partial^2 Y / \partial F_N{}^2 \Big|_{OA} = \right)$ son:

$\frac{2\,Fn\left(\frac{dp}{p} + \frac{dw}{w}\right)}{Fnn}$ $\qquad$ $\frac{2\left(\frac{dp}{p} + \frac{dw}{w}\right)}{Fnn}$

- $\left(\partial P / \partial F_{NN} \Big|_{OA} = \right)$ y $\left(\partial^2 P / \partial F_{NN}{}^2 \Big|_{OA} = \right)$son:

$-\frac{(dy - dk\,Fk\,p)}{Fn^2}$ $\qquad$ 0

- $\left(\partial Y / \partial F_{NN} \Big|_{OA} = \right)$ y $\left(\partial^2 Y / \partial F_{NN}{}^2 \Big|_{OA} = \right)$son:

$-\frac{Fn^2\left(\frac{dp}{p} + \frac{dw}{w}\right)}{Fnn^2}$ $\qquad$ $\frac{2\,Fn^2\left(\frac{dp}{p} + \frac{dw}{w}\right)}{Fnn^3}$

- $\left(\partial P / \partial F_K \Big|_{OA} = \right)$ y $\left(\partial^2 P / \partial F_K{}^2 \Big|_{OA} = \right)$ son:

$\frac{dk\,Fnn\,p}{Fn^2}$ $\qquad$ 0

- $\left(\partial Y / \partial F_K \Big|_{OA} = \right)$ y $\left(\partial^2 Y / \partial F_K{}^2 \Big|_{OA} = \right)$son:

dk $\qquad$ 0

Es interesante destacar los efectos que producen los cambios en la oferta de sus determinantes, pues estos producen lo que se conoce como **inflación por tirón de la demanda**.

Así, respectivamente, se obtiene:

- $\left(\partial P / \partial T \Big|_{DA} = \right)$ y $\left(\partial^2 P / \partial T^2 \Big|_{DA} = \right)$ son:

$-\frac{C1\,mn\,p^2}{(C2 + I3\ [illegible])}$ $\qquad$ 0

- $\left(\partial Y / \partial T\big|_{DA} = \right)$ y $\left(\partial^2 Y / \partial T^2\big|_{DA} = \right)$ son:

$$\frac{C1\ Fn\ mr}{1 + C1 \ldots r + I2\ mr - Fn\ (C2 + I3 \ldots)}$$ $$0$$

- $\left(\partial P / \partial M\big|_{DA} = \right)$ y $\left(\partial^2 P / \partial M^2\big|_{DA} = \right)$ son:

$$\frac{1}{M}$$ $$-\frac{1}{M^2}$$

- $\left(\partial Y / \partial M\big|_{DA} = \right)$ y $\left(\partial^2 Y / \partial M^2\big|_{DA} = \right)$ son:

$$\frac{Fn\ \ldots 2 + I3 \ldots}{1 + C1 \ldots + I2 \ldots + Fn\ (C2 + I3 \ldots)\ldots^2}$$ $$0$$

- $\left(\partial P / \partial \pi\big|_{DA} = \right)$ y $\left(\partial^2 P / \partial \pi^2\big|_{DA} = \right)$ son:

$$-\frac{mr\ p^2}{M}$$ $$0$$

- $\left(\partial Y / \partial \pi\big|_{DA} = \right)$ y $\left(\partial^2 Y / \partial \pi^2\big|_{DA} = \right)$ son:

$$-\frac{Fn\ (C2 + I3 \ldots)}{-\ldots 1 + C1 \ldots + I2 \ldots + Fn\ (C2 + I3 \ldots)}$$ $$0$$

- $\left(\partial P / \partial g\big|_{DA} = \right)$ y $\left(\partial^2 P / \partial g^2\big|_{DA} = \right)$ son:

$$\frac{mr\ p \ldots}{(C2 + I3 \ldots) M}$$ $$0$$

- $\left(\partial Y / \partial g\big|_{DA} = \right)$ y $\left(\partial^2 Y / \partial g^2\big|_{DA} = \right)$ son:

$$-\frac{Fn\ mr}{1 + C1 \ldots r + I2\ mr - Fn\ (C2 + I3 \ldots)}$$ $$0$$

Implementaciones en Matemática 4.1

CLASSIC MODEL

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Mathematica Initializations

Multiplicator Fuctions

```
Multiplicator[vardep_, varind_, name_, especif_] :=
 Module[{a}, a = D[vardep, varind]; If[Simplify[a > 0, especif], Print[name / varind == a > 0],
   If[Simplify[a < 0, especif], Print[name / varind == a < 0],
    If[Simplify[a == 0, especif], Print[name / varind == a]]],
   Print[name / varind == a → "Undefinide Sign"]]]
```

WHITOUT DICOTOMIC

GENERAL SOLUTIONS

```
Unprotect[N, C, In];
ClearAll[Y, WP, N, PC, In, R, dM, dB];
A =
 Simplify[Solve[{dy == Fk dk + Fn dN, dwp == Fnk dk + Fnn dN, dN == Np dwp, dM - M dp == p (mr dr + my dy),
     dy == dc + dI + dg + δ dk, dI == I1 dk + I2 dn + I3 (dr - d pi),
     dc == C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi)},
   {dy, dwp, dN, dp, dc, dI, dr}]];
A[[1, 1]]
A[[1, 2]]
A[[1, 3]]
A[[1, 4]]
A[[1, 5]]
A[[1, 6]]
A[[1, 7]]
A = {dy, dwp, dN, dp, dc, dI, dr} /. A;
```

$$dc \to -dg + dk\,Fk - dk\,I1 - dn\,I2 + \frac{dk\,Fn\,Fnk\,Np}{1 - Fnn\,Np} + d\,I3\,pi - dk\,\delta + \left(I3\,p\left(-\frac{C1\,dM\,(B+M)\,pi}{p^2} - \frac{dk\,(Fk + Fn\,Fnk\,Np - Fk\,Fnn\,Np)\,(B\,C1\,my\,pi + M\,(p - C1\,p + C1\,my\,pi))}{(-1 + Fnn\,Np)\,p} - M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta - \frac{C1\,(-B\,dpi - dpi\,M + dT\,p + dB\,pi + dM\,pi + dk\,p\,\delta)}{p}\right)\right)\right) \Big/ (-C2\,M\,p - I3\,M\,p + C1\,(B+M)\,mr\,pi)$$

$$dwp \to \frac{dk\,Fnk}{1 - Fnn\,Np}$$

$$dI \to dk\,I1 + dn\,I2 - d\,I3\,pi - \left(I3\,p\left(-\frac{C1\,dM\,(B+M)\,pi}{p^2} - \frac{dk\,(Fk + Fn\,Fnk\,Np - Fk\,Fnn\,Np)\,(B\,C1\,my\,pi + M\,(p - C1\,p + C1\,my\,pi))}{(-1 + Fnn\,Np)\,p} - M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta - \frac{C1\,(-B\,dpi - dpi\,M + dT\,p + dB\,pi + dM\,pi + dk\,p\,\delta)}{p}\right)\right)\right) \Big/ (-C2\,M\,p - I3\,M\,p + C1\,(B+M)\,mr\,pi)$$

$$dp \to \frac{1}{M}\left(dM + dk\,my\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)p + \left(mr\,p^2\left(-\frac{C1\,dM\,(B+M)\,pi}{p^2} - \frac{dk\,(Fk + Fn\,Fnk\,Np - Fk\,Fnn\,Np)\,(B\,C1\,my\,pi + M\,(p - C1\,p + C1\,my\,pi))}{(-1 + Fnn\,Np)\,p} - M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta - \frac{C1\,(-B\,dpi - dpi\,M + dT\,p + dB\,pi + dM\,pi + dk\,p\,\delta)}{p}\right)\right)\right) \Big/ (-C2\,M\,p - I3\,M\,p + C1\,(B+M)\,mr\,pi)\right)$$

$$dr \to \left(p\left(\frac{C1\,dM\,(B+M)\,pi}{p^2} + \frac{dk\,(Fk + Fn\,Fnk\,Np - Fk\,Fnn\,Np)\,(B\,C1\,my\,pi + M\,(p - C1\,p + C1\,my\,pi))}{(-1 + Fnn\,Np)\,p} + M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta - \frac{C1\,(-B\,dpi - dpi\,M + dT\,p + dB\,pi + dM\,pi + dk\,p\,\delta)}{p}\right)\right)\right) \Big/ (-C2\,M\,p - I3\,M\,p + C1\,(B+M)\,mr\,pi)$$

$$dy \to dk\left(Fk + \frac{Fn\,Fnk\,Np}{1 - Fnn\,Np}\right)$$

$$dN \to \frac{dk\,Fnk\,Np}{1 - Fnn\,Np}$$

STATIC COMPARATIVE

To calculate the sensibility of the endogenous variables (Y,C,N,w,r,In,P) with regarding the exogenous ones (dk,dg,dT,dpi,dM,dB) only press **" Shif + Enter "** in the following expression:

```
ClearAll[exogenas, endogenas];
Unprotected[dWP];
endogenas = {dY, dWP, dNN, dP, dC, dIn, dR};
exogenas = {dg, dk, dT, dM, dpi};
Do[Multiplicator[A[[1, j]], exogenas[[i]], endogenas[[j]],
Fnn < 0 && Fnk > 0 && Fk > 0 && C1 > 0 && C2 < 0 && I1 < 0 && I2 > 0 && I3 < 0 && Fkk < 0 && p > 0 && mr < 0 &&
my > 0 && Np > 0 && M > 0], {i, 1, Length[exogenas]}, {j, 1, Length[endogenas]}]
```

$$\frac{dY}{dg} == 0$$

$$\frac{dWP}{dg} == 0$$

$$\frac{dNN}{dg} == 0$$

$$\frac{dP}{dg} == -\frac{mr\, p^2}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dg} == -1 - \frac{I3\, M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dg} == \frac{I3\, M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dg} == \frac{M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dk} == Fk + \frac{Fn\, Fnk\, Np}{1 - Fnn\, Np} \rightarrow \text{Undefinide Sign}$$

$$\frac{dWP}{dk} == \frac{Fnk}{1 - Fnn\, Np} > 0$$

$$\frac{dNN}{dk} == \frac{Fnk\, Np}{1 - Fnn\, Np} > 0$$

$$\frac{dP}{dk} == \frac{my \left(-Fk + \frac{Fn\, Fnk\, Np}{-1 + Fnn\, Np}\right) p + \frac{mr\, p^2 \left(-\frac{(Fk + Fn\, Fnk\, Np - Fk\, Fnn\, Np)\, (B\, C1\, my\, pi + M\, (p - C1\, p + C1\, my\, pi))}{(-1 + Fnn\, Np)\, p} - M\, (I1 + \delta - C1\, \delta)\right)}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi}}{M} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dk} == Fk - I1 + \frac{Fn\, Fnk\, Np}{1 - Fnn\, Np} - \delta + \frac{I3\, p \left(-\frac{(Fk + Fn\, Fnk\, Np - Fk\, Fnn\, Np)\, (B\, C1\, my\, pi + M\, (p - C1\, p + C1\, my\, pi))}{(-1 + Fnn\, Np)\, p} - M\, (I1 + \delta - C1\, \delta)\right)}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow$$

Undefinide Sign

$$\frac{dIn}{dk} == I1 - \frac{I3\, p \left(-\frac{(Fk + Fn\, Fnk\, Np - Fk\, Fnn\, Np)\, (B\, C1\, my\, pi + M\, (p - C1\, p + C1\, my\, pi))}{(-1 + Fnn\, Np)\, p} - M\, (I1 + \delta - C1\, \delta)\right)}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dk} == \frac{p \left(\frac{(Fk + Fn\, Fnk\, Np - Fk\, Fnn\, Np)\, (B\, C1\, my\, pi + M\, (p - C1\, p + C1\, my\, pi))}{(-1 + Fnn\, Np)\, p} + M\, (I1 + \delta - C1\, \delta)\right)}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dT} == 0$$

$$\frac{dWP}{dT} == 0$$

$$\frac{dNN}{dT} == 0$$

$$\frac{dP}{dT} == \frac{C1\, mr\, p^2}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dT} == \frac{C1\, I3\, M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dT} == -\frac{C1\, I3\, M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dT} == -\frac{C1\, M\, p}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dM} == 0$$

$$\frac{dWP}{dM} == 0$$

$$\frac{dNN}{dM} == 0$$

$$\frac{dP}{dM} == \frac{1 + \frac{mr\, p^2 \left(-\frac{C1\, (B + M)\, pi}{p^2} + \frac{C1\, M\, pi}{p}\right)}{-C2\, M\, p - I3\, M\, p + C1\, (B + M)\, mr\, pi}}{M} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dM} == \frac{I3\ p\left(-\frac{C1\,(B+M)\,pi}{p^2}+\frac{C1\,M\,pi}{p}\right)}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dM} == -\frac{I3\ p\left(-\frac{C1\,(B+M)\,pi}{p^2}+\frac{C1\,M\,pi}{p}\right)}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dR}{dM} == \frac{p\left(\frac{C1\,(B+M)\,pi}{p^2}-\frac{C1\,M\,pi}{p}\right)}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dY}{dpi} == 0$$

$$\frac{dWP}{dpi} == 0$$

$$\frac{dNN}{dpi} == 0$$

$$\frac{dP}{dpi} == -\frac{mr\left(-C2-\frac{C1\,(-B-M)}{p}\right)p^2}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dC}{dpi} == -\frac{I3\,M\left(-C2-\frac{C1\,(-B-M)}{p}\right)p}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dpi} == \frac{I3\,M\left(-C2-\frac{C1\,(-B-M)}{p}\right)p}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

$$\frac{dR}{dpi} == \frac{M\left(-C2-\frac{C1\,(-B-M)}{p}\right)p}{-C2\,M\,p-I3\,M\,p+C1\,(B+M)\,mr\,pi} \to \text{Undefinide Sign}$$

IS-LM AND *DA-OA* ANALYSIS

IS EQUATION

```
IS =
 FullSimplify[
  Solve[
   dy == ReplaceAll[ReplaceAll[dc + dI + dg + δ dk,
      {dc -> C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi),
       dI -> I1 dk + I2 dn + I3 (dr - d pi)}], {dk -> 0}], dr]]
```

$$\left\{\left\{dr \to \frac{-dg+C2\,dpi+dy-dn\,I2+d\,I3\,pi-C1\left(-dT+dy+\frac{dpi\,(B+M)}{p}+\frac{(dp\,(B+M)-(dB+dM)\,p)\,pi}{p^2}\right)}{C2+I3}\right\}\right\}$$

```
IS = dr /. IS;
FullSimplify[D[IS, dy]]
```

$$\left\{\frac{1-C1}{C2+I3}\right\}$$

LM EQUATION

```
LM = FullSimplify[Solve[dM - M dp == p (mr dr + my dy), dr]]
```

$$\left\{\left\{dr \to \frac{dM-dp\,M-dy\,my\,p}{mr\,p}\right\}\right\}$$

```
LM = dr /. LM;
D[LM, dy]
```

```
{-my/mr}
```

DA EQUATION

```
DA = FullSimplify[Solve[IS == LM, dp]]
{{dp →
  -(p (-dM I3 + (C1 (dT - dy) + dy - dn I2) mr p + dy I3 my p - mr (C1 dpi (B + M) + dg p) + C2 (-dM + dpi mr p + dy my p) +
       C1 dB mr pi + C1 dM mr pi + d I3 mr p pi)) / ((C2 + I3) M p - C1 (B + M) mr pi)}}
DA = dp /. DA;
FullSimplify[D[DA, dy]]
```

$$\left\{\frac{((-1+C1)\,mr-(C2+I3)\,my)\,p^2}{(C2+I3)\,M\,p-C1\,(B+M)\,mr\,pi}\right\}$$

OA EQUATION

$$OA = dk\left(Fk+\frac{Fn\,Fnk\,Np}{1-Fnn\,Np}\right)$$

$$dk\left(Fk+\frac{Fn\,Fnk\,Np}{1-Fnn\,Np}\right)$$

```
D[OA, dp]
0
```

WHIT DICOTOMIC

GENERAL SOLUTIONS

```
Unprotect[N, C, In];
ClearAll[Y, WP, N, PC, In, R, dM, dB];
dB = -dM;
A =
 Simplify[Solve[{dy == Fk dk + Fn dN, dwp == Fnk dk + Fnn dN, dN == Np dwp, dM - M dp == p (mr dr + my dy),
    dy == dc + dI + dg + δ dk, dI == I1 dk + I2 dn + I3 (dr - d pi),
    dc == C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi)},
   {dy, dwp, dN, dp, dc, dI, dr}]];
A[[1, 1]]
A[[1, 2]]
A[[1, 3]]
A[[1, 4]]
A[[1, 5]]
A[[1, 6]]
A[[1, 7]]
A = {dy, dwp, dN, dp, dc, dI, dr} /. A;
```

$$dc \to -dg + dk\,Fk - dk\,I1 - dn\,I2 + \frac{dk\,Fn\,Fnk\,Np}{1 - Fnn\,Np} + d\,I3\,pi - dk\,\delta - \left(I3\left(\frac{C1\,dM\,(B+M)\,pi}{p^2} + dk\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)\left(M - C1\,M + \frac{C1\,(B+M)\,my\,pi}{p}\right) + M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta + C1\left(-dT + \frac{dpi\,(B+M)}{p} - dk\,\delta\right)\right)\right)\right) \Big/ \left(-(C2 + I3)\,M + \frac{C1\,(B+M)\,mr\,pi}{p}\right)$$

$$dwp \to \frac{dk\,Fnk}{1 - Fnn\,Np}$$

$$dI \to dk\,I1 + dn\,I2 - d\,I3\,pi + \left(I3\left(\frac{C1\,dM\,(B+M)\,pi}{p^2} + dk\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)\left(M - C1\,M + \frac{C1\,(B+M)\,my\,pi}{p}\right) + M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta + C1\left(-dT + \frac{dpi\,(B+M)}{p} - dk\,\delta\right)\right)\right)\right) \Big/ \left(-(C2 + I3)\,M + \frac{C1\,(B+M)\,mr\,pi}{p}\right)$$

$$dp \to \frac{1}{M}\left(dM + dk\,my\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)p - \left(mr\,p\left(\frac{C1\,dM\,(B+M)\,pi}{p^2} + dk\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)\left(M - C1\,M + \frac{C1\,(B+M)\,my\,pi}{p}\right) + M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta + C1\left(-dT + \frac{dpi\,(B+M)}{p} - dk\,\delta\right)\right)\right)\right) \Big/ \left(-(C2 + I3)\,M + \frac{C1\,(B+M)\,mr\,pi}{p}\right)\right)$$

$$dr \to \left(\frac{C1\,dM\,(B+M)\,pi}{p^2} + dk\left(-Fk + \frac{Fn\,Fnk\,Np}{-1 + Fnn\,Np}\right)\left(M - C1\,M + \frac{C1\,(B+M)\,my\,pi}{p}\right) + M\left(dg - C2\,dpi + dk\,I1 + dn\,I2 - d\,I3\,pi + dk\,\delta + C1\left(-dT + \frac{dpi\,(B+M)}{p} - dk\,\delta\right)\right)\right) \Big/ \left(-(C2 + I3)\,M + \frac{C1\,(B+M)\,mr\,pi}{p}\right)$$

$$dy \to dk\left(Fk + \frac{Fn\,Fnk\,Np}{1 - Fnn\,Np}\right)$$

$$dN \to \frac{dk\,Fnk\,Np}{1 - Fnn\,Np}$$

STATIC COMPARATIVE

To calculate the sensibility of the endogenous variables (Y,C,N,w,r,In,P) with regarding the exogenous ones (dk,dg,dT,dpi,dM,dB) only press **" Shif + Enter "** in the following expression:

```
ClearAll[exogenas, endogenas];
endogenas = {dY, dWP, dNN, dP, dC, dIn, dR};
exogenas = {dg, dk, dT, dM, dpi};
Do[Multiplicator[A[[1, j]], exogenas[[i]], endogenas[[j]],
Fnn < 0 && Fnk > 0 && Fk > 0 && C1 > 0 && C2 < 0 && I1 < 0 && I2 > 0 && I3 < 0 && Fkk < 0 && p > 0 && mr < 0 &&
my > 0 && Np > 0 && M > 0], {i, 1, Length[exogenas]}, {j, 1, Length[endogenas]}]
```

$$\frac{dY}{dg} == 0$$

$$\frac{dWP}{dg} == 0$$

$$\frac{dNN}{dg} == 0$$

$$\frac{dP}{dg} == -\frac{mr\,p}{-(C2 + I3)\,M + \frac{C1\,(B+M)\,mr\,pi}{p}} \to \text{Undefinide Sign}$$

$$\frac{dC}{dg} == -1 - \frac{I3\, M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dg} == \frac{I3\, M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dg} == \frac{M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dk} == Fk + \frac{Fn\, Fnk\, Np}{1 - Fnn\, Np} \rightarrow \text{Undefinide Sign}$$

$$\frac{dWP}{dk} == \frac{Fnk}{1 - Fnn\, Np} > 0$$

$$\frac{dNN}{dk} == \frac{Fnk\, Np}{1 - Fnn\, Np} > 0$$

$$\frac{dP}{dk} == \frac{my\left(-Fk + \frac{Fn\, Fnk\, Np}{-1+Fnn\, Np}\right) p - \frac{mr\, p\left(\left(-Fk+\frac{Fn\, Fnk\, Np}{-1+Fnn\, Np}\right)\left(M - C1\, M + \frac{C1\,(B+M)\, my\, pi}{p}\right) + M\,(I1 + \delta - C1\,\delta)\right)}{-(C2+I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}}}{M} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dk} == Fk - I1 + \frac{Fn\, Fnk\, Np}{1 - Fnn\, Np} - \delta - \frac{I3\left(\left(-Fk + \frac{Fn\, Fnk\, Np}{-1+Fnn\, Np}\right)\left(M - C1\, M + \frac{C1\,(B+M)\, my\, pi}{p}\right) + M\,(I1 + \delta - C1\,\delta)\right)}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dk} == I1 + \frac{I3\left(\left(-Fk + \frac{Fn\, Fnk\, Np}{-1+Fnn\, Np}\right)\left(M - C1\, M + \frac{C1\,(B+M)\, my\, pi}{p}\right) + M\,(I1 + \delta - C1\,\delta)\right)}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dk} == \frac{\left(-Fk + \frac{Fn\, Fnk\, Np}{-1+Fnn\, Np}\right)\left(M - C1\, M + \frac{C1\,(B+M)\, my\, pi}{p}\right) + M\,(I1 + \delta - C1\,\delta)}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dT} == 0$$

$$\frac{dWP}{dT} == 0$$

$$\frac{dNN}{dT} == 0$$

$$\frac{dP}{dT} == \frac{C1\, mr\, p}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dT} == \frac{C1\, I3\, M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dT} == -\frac{C1\, I3\, M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dT} == -\frac{C1\, M}{-(C2 + I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dM} == 0$$

$$\frac{dWP}{dM} == 0$$

$$\frac{dNN}{dM} == 0$$

$$\frac{dP}{dM} == \frac{1 - \frac{C1\,(B+M)\, mr\, pi}{p\left(-(C2+I3)\, M + \frac{C1\,(B+M)\, mr\, pi}{p}\right)}}{M} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dM} == -\frac{C1\ I3\ (B+M)\ pi}{p^2 \left(-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}\right)} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dM} == \frac{C1\ I3\ (B+M)\ pi}{p^2 \left(-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}\right)} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dM} == \frac{C1\ (B+M)\ pi}{p^2 \left(-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}\right)} \rightarrow \text{Undefinide Sign}$$

$$\frac{dY}{dpi} == 0$$

$$\frac{dWP}{dpi} == 0$$

$$\frac{dNN}{dpi} == 0$$

$$\frac{dP}{dpi} == -\frac{mr \left(-C2 + \frac{C1\ (B+M)}{p}\right) p}{-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dpi} == -\frac{I3\ M \left(-C2 + \frac{C1\ (B+M)}{p}\right)}{-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dpi} == \frac{I3\ M \left(-C2 + \frac{C1\ (B+M)}{p}\right)}{-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dpi} == \frac{M \left(-C2 + \frac{C1\ (B+M)}{p}\right)}{-(C2+I3)\ M + \frac{C1\ (B+M)\ mr\ pi}{p}} \rightarrow \text{Undefinide Sign}$$

IS-LM AND *DA-OA* ANALYSIS

IS EQUATION

```
dB = -dM;
B = -M;
IS =
 FullSimplify[
  Solve[
   dy == ReplaceAll[ReplaceAll[dc + dI + dg + δ dk,
      {dc -> C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi),
       dI -> I1 dk + I2 dn + I3 (dr - d pi)}], {dk -> 0}], dr]]
```

$$\left\{\left\{dr \rightarrow \frac{-dg + C2\ dpi + C1\ dT + dy - C1\ dy - dn\ I2 + d\ I3\ pi}{C2 + I3}\right\}\right\}$$

```
dB = -dM;
B = -M;
IS = dr /. IS;
FullSimplify[D[IS, dy]]
```

$$\left\{\frac{1 - C1}{C2 + I3}\right\}$$

LM EQUATION

```
dB = -dM;
B = -M;
LM = FullSimplify[Solve[dM - M dp == p (mr dr + my dy), dr]]
```

$$\left\{\left\{dr \to \frac{dM - dp\,M - dy\,my\,p}{mr\,p}\right\}\right\}$$

```
dB = -dM;
B = -M;
LM = dr /. LM;
D[LM, dy]
```

$$\left\{-\frac{my}{mr}\right\}$$

DA EQUATION

```
dB = -dM;
B = -M;
DA = FullSimplify[Solve[IS == LM, dp]]
```

$$\left\{\left\{dp \to -\frac{mr\,p\left(\frac{-dM + dy\,my\,p}{mr\,p} + \frac{-dg + C2\,dpi + C1\,dT + dy - C1\,dy - dn\,I2 + d\,I3\,pi}{C2 + I3}\right)}{M}\right\}\right\}$$

```
dB = -dM;
B = -M;
DA = dp /. DA;
FullSimplify[D[DA, dy]]
```

$$\left\{\frac{((-1 + C1)\,mr - (C2 + I3)\,my)\,p}{(C2 + I3)\,M}\right\}$$

OA EQUATION

$$OA = dk\left(Fk + \frac{Fn\,Fnk\,Np}{1 - Fnn\,Np}\right)$$

$$dk\left(Fk + \frac{Fn\,Fnk\,Np}{1 - Fnn\,Np}\right)$$

```
D[OA, dp]
```

0

KEYNESIAN MODEL

Authors: Jorge Mauricio Oviedo
Victor Manuel Poma

Mathematica Initializations

Multiplicator Fuctions

```
Multiplicator[vardep_, varind_, name_, especif_] :=
 Module[{a}, a = D[vardep, varind]; If[Simplify[a > 0, especif], Print[name / varind == a > 0],
   If[Simplify[a < 0, especif], Print[name / varind == a < 0],
    If[Simplify[a == 0, especif], Print[name / varind == a]]],
   Print[name / varind == a → "Undefinide Sign"]]]
```

WHITOUT DICOTOMIC

dB = - dM

GENERAL SOLUTIONS

```
Unprotect[N, C, In];
ClearAll[Y, N, P, C, In, R, w, dM, dB];
dB = -dM;
B = -M;
A =
 Simplify[Solve[{dy == Fk dk + Fn dN, (dw p - w dp ) / p ^2 == Fnk dk + Fnn dN, dM - M dp / p == p (mr dr + my dy),
    dy == dc + dIn + dg + δ dk, dIn == I1 dk + I2 dN + I3 (dr - dpi),
    dc == C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi)},
   {dy, dN, dp, dc, dIn, dr}]];
A[[1, 1]]
A[[1, 2]]
A[[1, 3]]
A[[1, 4]]
A[[1, 5]]
A[[1, 6]]
A = {dy, dN, dp, dc, dIn, dr} /. A;
dc →
 (C1 (dw Fn I3 M - dT Fnn I3 M p - dM Fn I3 w - dg Fn mr p w + dT Fn mr p w - dT I2 mr p w + dpi Fn I3 mr p w + dT Fn I3 my p w -
       dk p (-(Fnn I3 M + I2 mr w) (Fk - δ) + Fn (Fnk I3 M + I1 mr w - I3 my w δ))) +
    C2 (dw (Fn - I2) M - dg Fnn M p - dM Fn w + dM I2 w + dpi Fn mr p w - dpi I2 mr p w + dg Fn my p w +
       dk p (Fk Fnn M - Fnn I1 M + Fnk I2 M - Fk I2 my w - Fnn M δ + Fn (-Fnk M + my w (I1 + δ))))) /
  (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
dIn → (-(-1 + C1) dw Fn I3 M - dk Fn Fnk I3 M p + C1 dk Fn Fnk I3 M p - dg Fnn I3 M p + C1 dT Fnn I3 M p + dk Fk Fnn I3 M p -
    C1 dk Fk Fnn I3 M p - dM Fn I3 w + C1 dM Fn I3 w - dk Fn I1 mr p w + C1 dk Fn I1 mr p w - dg I2 mr p w +
    C1 dT I2 mr p w + dk Fk I2 mr p w - C1 dk Fk I2 mr p w + dpi Fn I3 mr p w - C1 dpi Fn I3 mr p w + dg Fn I3 my p w -
    C1 dT Fn I3 my p w + C2 (dw I2 M + I2 (-dM + dpi mr p) w + dk p (Fnn I1 M - Fnk I2 M - Fn I1 my w + Fk I2 my w)) -
    dk Fnn I3 M p δ + C1 dk Fnn I3 M p δ - dk I2 mr p w δ + C1 dk I2 mr p w δ + dk Fn I3 my p w δ - C1 dk Fn I3 my p w δ) /
  (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
```

$$dp \to \frac{1}{w}\left(p^2\left(-dk\,Fnk + \frac{dw}{p} + \left(Fnn\,(dk\,Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + (C2+I3)\,(-dw\,M + dk\,Fnk\,M\,p + dM\,w) + mr\,p\,w\,(dg - C2\,dpi - C1\,dT + dk\,I1 - dpi\,I3 + dk\,\delta - C1\,dk\,\delta))\right) / \left(p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))\right)\right)\right)$$

$$dr \to (-dw\,((-1+C1)\,Fn + I2)\,M - dg\,Fnn\,M\,p + C2\,dpi\,Fnn\,M\,p + C1\,dT\,Fnn\,M\,p + dpi\,Fnn\,I3\,M\,p - dM\,Fn\,w + C1\,dM\,Fn\,w + dM\,I2\,w + dg\,Fn\,my\,p\,w - C2\,dpi\,Fn\,my\,p\,w - C1\,dT\,Fn\,my\,p\,w - dpi\,Fn\,I3\,my\,p\,w + dk\,p\,(-Fk\,((-1+C1)\,Fnn\,M + I2\,my\,w) + M\,(Fnk\,I2 - Fnn\,(I1 + \delta - C1\,\delta)) + Fn\,((-1+C1)\,Fnk\,M + my\,w\,(I1 + \delta - C1\,\delta)))) / (p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)))$$

$$dy \to dk\,Fk - (Fn\,(dk\,Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + (C2+I3)\,(-dw\,M + dk\,Fnk\,M\,p + dM\,w) + mr\,p\,w\,(dg - C2\,dpi - C1\,dT + dk\,I1 - dpi\,I3 + dk\,\delta - C1\,dk\,\delta))) / (p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)))$$

$$dN \to -(dk\,Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + (C2+I3)\,(-dw\,M + dk\,Fnk\,M\,p + dM\,w) + mr\,p\,w\,(dg - C2\,dpi - C1\,dT + dk\,I1 - dpi\,I3 + dk\,\delta - C1\,dk\,\delta)) / (p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)))$$

STATIC COMPARATIVE

To calculate the sensibility of the endogenous variables (Y,C,N,r,In,P) with regarding the exogenous ones (dk,dg,dT,dpi,dM,dB,dW) only press **" Shif + Enter "** in the following expression:

```
ClearAll[exogenas, endogenas];
Unprotected[dWP];
endogenas = {dY, dNN, dP, dC, dIn, dR};
exogenas = {dg, dk, dT, dM, dpi, dw};
Do[Multiplicator[A[[1, j]], exogenas[[i]], endogenas[[j]],
Fnn < 0 && Fnk > 0 && Fk > 0 && C1 > 0 && C2 < 0 && I1 < 0 && I2 > 0 && I3 < 0 && Fkk < 0 && Fn > 0 && p > 0 &&
    w > 0 && mr < 0 &&
my > 0 && Mp > 0 && M > 0], {i, 1, Length[exogenas]}, {j, 1, Length[endogenas]}]
```

$$\frac{dY}{dg} == -\frac{Fn\,mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dg} == -\frac{mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dP}{dg} == \frac{Fnn\,mr\,p^2}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dC}{dg} == \frac{-C1\,Fn\,mr\,p\,w + C2\,(-Fnn\,M\,p + Fn\,my\,p\,w)}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dg} == \frac{-Fnn\,I3\,M\,p - I2\,mr\,p\,w + Fn\,I3\,my\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dR}{dg} == \frac{-Fnn\,M\,p + Fn\,my\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dY}{dk} == Fk - \frac{Fn\,(Fnk\,(C2+I3)\,M\,p + Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + mr\,p\,w\,(I1 + \delta - C1\,\delta))}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dk} == -\frac{Fnk\,(C2+I3)\,M\,p + Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + mr\,p\,w\,(I1 + \delta - C1\,\delta)}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dP}{dk} == \frac{p^2 \left(-Fnk + \frac{Fnn\,(Fnk\,(C2+I3)\,M\,p + Fk\,((-1+C1)\,mr - (C2+I3)\,my)\,p\,w + mr\,p\,w\,(I1+\delta - C1\,\delta))}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))}\right)}{w} \to \text{Undefinide Sign}$$

$$\frac{dC}{dk} == (-C1\,p\,(-(Fnn\,I3\,M + I2\,mr\,w)\,(Fk - \delta) + Fn\,(Fnk\,I3\,M + I1\,mr\,w - I3\,my\,w\,\delta)) + C2\,p\,(Fk\,Fnn\,M - Fnn\,I1\,M + Fnk\,I2\,M - Fk\,I2\,my\,w - Fnn\,M\,\delta + Fn\,(-Fnk\,M + my\,w\,(I1+\delta))))\;/\;(p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))) \to \text{Undefinide Sign}$$

$$\frac{dIn}{dk} == (-Fn\,Fnk\,I3\,M\,p + C1\,Fn\,Fnk\,I3\,M\,p + Fk\,Fnn\,I3\,M\,p - C1\,Fk\,Fnn\,I3\,M\,p - Fn\,I1\,mr\,p\,w + C1\,Fn\,I1\,mr\,p\,w + Fk\,I2\,mr\,p\,w - C1\,Fk\,I2\,mr\,p\,w + C2\,p\,(Fnn\,I1\,M - Fnk\,I2\,M - Fn\,I1\,my\,w + Fk\,I2\,my\,w) - Fnn\,I3\,M\,p\,\delta + C1\,Fnn\,I3\,M\,p\,\delta - I2\,mr\,p\,w\,\delta + C1\,I2\,mr\,p\,w\,\delta + Fn\,I3\,my\,p\,w\,\delta - C1\,Fn\,I3\,my\,p\,w\,\delta)\;/\;(p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))) \to \text{Undefinide Sign}$$

$$\frac{dR}{dk} == (-Fk\,((-1+C1)\,Fnn\,M + I2\,my\,w) + M\,(Fnk\,I2 - Fnn\,(I1+\delta - C1\,\delta)) + Fn\,((-1+C1)\,Fnk\,M + my\,w\,(I1+\delta - C1\,\delta)))\;/\;(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)) \to \text{Undefinide Sign}$$

$$\frac{dY}{dT} == \frac{C1\,Fn\,mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dT} == \frac{C1\,mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dP}{dT} == -\frac{C1\,Fnn\,mr\,p^2}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dC}{dT} == \frac{C1\,(-Fnn\,I3\,M\,p + Fn\,mr\,p\,w - I2\,mr\,p\,w + Fn\,I3\,my\,p\,w)}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dT} == \frac{C1\,Fnn\,I3\,M\,p + C1\,I2\,mr\,p\,w - C1\,Fn\,I3\,my\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dR}{dT} == \frac{C1\,Fnn\,M\,p - C1\,Fn\,my\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dY}{dM} == -\frac{Fn\,(C2+I3)\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dM} == -\frac{(C2+I3)\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dP}{dM} == \frac{Fnn\,(C2+I3)\,p}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dC}{dM} == \frac{-C1\,Fn\,I3\,w + C2\,(-Fn\,w + I2\,w)}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dM} == \frac{-C2\,I2\,w - Fn\,I3\,w + C1\,Fn\,I3\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dR}{dM} == \frac{-Fn\,w + C1\,Fn\,w + I2\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dY}{dpi} == -\frac{Fn\,(-C2 - I3)\,mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dpi} == -\frac{(-C2 - I3)\,mr\,w}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dP}{dpi} == \frac{Fnn\,(-C2 - I3)\,mr\,p^2}{Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w)} \to \text{Undefinide Sign}$$

$$\frac{dC}{dpi} == \frac{C1\,Fn\,I3\,mr\,p\,w + C2\,(Fn\,mr\,p\,w - I2\,mr\,p\,w)}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dIn}{dpi} == \frac{C2\,I2\,mr\,p\,w + Fn\,I3\,mr\,p\,w - C1\,Fn\,I3\,mr\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dR}{dpi} == \frac{C2\,Fnn\,M\,p + Fnn\,I3\,M\,p - C2\,Fn\,my\,p\,w - Fn\,I3\,my\,p\,w}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dY}{dw} == \frac{Fn\,(C2+I3)\,M}{p\,(Fnn\,I3\,M + (-Fn\,mr + C1\,Fn\,mr + I2\,mr - Fn\,I3\,my)\,w + C2\,(Fnn\,M - Fn\,my\,w))} \to \text{Undefinide Sign}$$

$$\frac{dNN}{dw} == \frac{(C2 + I3)\, M}{p\, (Fnn\, I3\, M + (-Fn\, mr + C1\, Fn\, mr + I2\, mr - Fn\, I3\, my)\, w + C2\, (Fnn\, M - Fn\, my\, w))} \rightarrow \text{Undefinide Sign}$$

$$\frac{dP}{dw} == \frac{p^2 \left(\frac{1}{p} - \frac{Fnn\, (C2+I3)\, M}{p\, (Fnn\, I3\, M+(-Fn\, mr+C1\, Fn\, mr+I2\, mr-Fn\, I3\, my)\, w+C2\, (Fnn\, M-Fn\, my\, w))}\right)}{w} \rightarrow \text{Undefinide Sign}$$

$$\frac{dC}{dw} == \frac{C2\, (Fn - I2)\, M + C1\, Fn\, I3\, M}{p\, (Fnn\, I3\, M + (-Fn\, mr + C1\, Fn\, mr + I2\, mr - Fn\, I3\, my)\, w + C2\, (Fnn\, M - Fn\, my\, w))} \rightarrow \text{Undefinide Sign}$$

$$\frac{dIn}{dw} == \frac{C2\, I2\, M - (-1 + C1)\, Fn\, I3\, M}{p\, (Fnn\, I3\, M + (-Fn\, mr + C1\, Fn\, mr + I2\, mr - Fn\, I3\, my)\, w + C2\, (Fnn\, M - Fn\, my\, w))} \rightarrow \text{Undefinide Sign}$$

$$\frac{dR}{dw} == -\frac{((-1 + C1)\, Fn + I2)\, M}{p\, (Fnn\, I3\, M + (-Fn\, mr + C1\, Fn\, mr + I2\, mr - Fn\, I3\, my)\, w + C2\, (Fnn\, M - Fn\, my\, w))} \rightarrow \text{Undefinide Sign}$$

IS-LM AND *DA-OA* ANALYSIS

IS EQUATION

```
dB = - dM;
B = - M;
IS =
 FullSimplify[
  Solve[
   dy ==
    ReplaceAll[
     ReplaceAll[ReplaceAll[ReplaceAll[dc + dIn + dg + δ dk,
        {dc -> C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi),
         dIn -> I1 dk + I2 dN + I3 (dr - dpi)}], {dk -> 0}], {dp / p → dw / w - Fnn dy / Fn^2}], {dN -> dy / Fn}], dr]]
```

$$\left\{\left\{dr \rightarrow dpi + \frac{-dg\, Fn + (C1\, dT + dy - C1\, dy)\, Fn - dy\, I2}{Fn\, (C2 + I3)}\right\}\right\}$$

```
IS = dr /. IS;
FullSimplify[D[IS, dy]]
```

$$\left\{\frac{Fn - C1\, Fn - I2}{C2\, Fn + Fn\, I3}\right\}$$

LM EQUATION

```
LM = FullSimplify[Solve[dy == ReplaceAll[((dM - M dp) / p^2 - mr dr) / my , {dp / p → dw / w - Fnn dy / Fn^2}], dr]]
```

$$\left\{\left\{dr \rightarrow \frac{dM - dp\, M - dy\, my\, p^2}{mr\, p^2}\right\}\right\}$$

```
LM = dr /. LM;
D[LM, dy]
```

$$\left\{-\frac{my}{mr}\right\}$$

DA EQUATION

```
dB = - dM; B = - M;
DA = FullSimplify[Solve[IS == LM, dp]]
```

$$\left\{\left\{dp \to -\frac{mr\, p^2 \left(dpi + \frac{-dg\, Fn + C1\, (dT - dy)\, Fn + dy\, (Fn - I2)}{Fn\, (C2 + I3)} + \frac{-dM + dy\, my\, p^2}{mr\, p^2}\right)}{M}\right\}\right\}$$

```
DA = dp /. DA;
FullSimplify[D[DA, dy]]
```

$$\left\{\frac{((-1 + C1)\, Fn\, mr + I2\, mr - Fn\, (C2 + I3)\, my)\, p^2}{Fn\, (C2 + I3)\, M}\right\}$$

OA EQUATION

$$OA = dk\left(Fk + \frac{Fn\, Fnk\, Ns}{1 - Fnn\, Ns}\right)$$

$$dk\left(Fk + \frac{Fn\, Fnk\, Ns}{1 - Fnn\, Ns}\right)$$

```
D[OA, dp]
0
```

WHIT DICOTOMIC

Supuesto dB distinto - dM

GENERAL SOLUTIONS

```
Unprotect[N, C, In];
ClearAll[Y, N, P, C, In, R, w, dM, dB];
B = - M;
A =
 Simplify[Solve[{dy == Fk dk + Fn dN, (dw p - w dp ) / p ^2 == Fnk dk + Fnn dN, dM - M dp / p == p (mr dr + my dy),
    dy == dc + dIn + dg + δ dk, dIn == I1 dk + I2 dN + I3 (dr - dpi),
    dc == C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p ^2) + dpi (M + B) / p) + C2 (dr - dpi)},
   {dy, dN, dp, dc, dIn, dr}]];
A[[1, 1]]
A[[1, 2]]
A[[1, 3]]
A[[1, 4]]
A[[1, 5]]
A[[1, 6]]
A = {dy, dN, dp, dc, dIn, dr} /. A;
dc →
 (C1 (dw Fn I3 M - dT Fnn I3 M p - dM Fn I3 w - dg Fn mr p w + dT Fn mr p w - dT I2 mr p w + dpi Fn I3 mr p w + dT Fn I3 my p w -
       dk p (-(Fnn I3 M + I2 mr w) (Fk - δ) + Fn (Fnk I3 M + I1 mr w - I3 my w δ))) +
    C2 (dw (Fn - I2) M - dg Fnn M p - dM Fn w + dM I2 w + dpi Fn mr p w - dpi I2 mr p w + dg Fn my p w +
       dk p (Fk Fnn M - Fnn I1 M + Fnk I2 M - Fk I2 my w - Fnn M δ + Fn (-Fnk M + my w (I1 + δ))))) /
  (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
```

```
dIn → (-(-1 + C1) dw Fn I3 M - dk Fn Fnk I3 M p + C1 dk Fn Fnk I3 M p - dg Fnn I3 M p + C1 dT Fnn I3 M p + dk Fk Fnn I3 M p -
    C1 dk Fk Fnn I3 M p - dM Fn I3 w + C1 dM Fn I3 w - dk Fn I1 mr p w + C1 dk Fn I1 mr p w - dg I2 mr p w +
    C1 dT I2 mr p w + dk Fk I2 mr p w - C1 dk Fk I2 mr p w + dpi Fn I3 mr p w - C1 dpi Fn I3 mr p w + dg Fn I3 my p w -
    C1 dT Fn I3 my p w + C2 (dw I2 M + I2 (-dM + dpi mr p) w + dk p (Fnn I1 M - Fnk I2 M - Fn I1 my w + Fk I2 my w)) -
    dk Fnn I3 M p δ + C1 dk Fnn I3 M p δ - dk I2 mr p w δ + C1 dk I2 mr p w δ + dk Fn I3 my p w δ - C1 dk Fn I3 my p w δ) /
  (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
dp →
 1/w (p^2 (-dk Fnk + dw/p + (Fnn (dk Fk ((-1 + C1) mr - (C2 + I3) my) p w + (C2 + I3) (-dw M + dk Fnk M p + dM w) + mr p w (dg -
          C2 dpi - C1 dT + dk I1 - dpi I3 + dk δ - C1 dk δ))) /
      (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))))
dr → (-dw ((-1 + C1) Fn + I2) M - dg Fnn M p + C2 dpi Fnn M p + C1 dT Fnn M p + dpi Fnn I3 M p -
    dM Fn w + C1 dM Fn w + dM I2 w + dg Fn my p w - C2 dpi Fn my p w - C1 dT Fn my p w - dpi Fn I3 my p w + dk p
     (-Fk ((-1 + C1) Fnn M + I2 my w) + M (Fnk I2 - Fnn (I1 + δ - C1 δ)) + Fn ((-1 + C1) Fnk M + my w (I1 + δ - C1 δ)))) /
  (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
dy → dk Fk - (Fn (dk Fk ((-1 + C1) mr - (C2 + I3) my) p w +
        (C2 + I3) (-dw M + dk Fnk M p + dM w) + mr p w (dg - C2 dpi - C1 dT + dk I1 - dpi I3 + dk δ - C1 dk δ))) /
   (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
dN → -(dk Fk ((-1 + C1) mr - (C2 + I3) my) p w +
      (C2 + I3) (-dw M + dk Fnk M p + dM w) + mr p w (dg - C2 dpi - C1 dT + dk I1 - dpi I3 + dk δ - C1 dk δ)) /
   (p (Fnn I3 M + (-Fn mr + C1 Fn mr + I2 mr - Fn I3 my) w + C2 (Fnn M - Fn my w)))
```

STATIC COMPARATIVE

To calculate the sensibility of the endogenous variables (Y,C,N,WP,R,In,P) with regarding the exogenous ones (dk,dg,dT,dpi,dM,dB) alone replaces them in the following expression

```
ClearAll[exogenas, endogenas];
Unprotected[dWP];
endogenas = {dY, dNN, dP, dC, dIn, dR};
exogenas = {dg, dk, dT, dM, dpi, dw};
Do[Multiplicator[A[[1, j]], exogenas[[i]], endogenas[[j]],
Fnn < 0 && Fnk > 0 && Fk > 0 && C1 > 0 && C2 < 0 && I1 < 0 && I2 > 0 && I3 < 0 && Fkk < 0 && p > 0 && mr < 0 &&
my > 0 && Mp > 0 && M > 0], {i, 1, Length[exogenas]}, {j, 1, Length[endogenas]}]
```

```
                    ((-1 + C1) Fn + I2) w
─────────────────────────────────────────────────────────
Fnn (C2 + I3) M p + ((-1 + C1) Fn mr + I2 mr - Fn (C2 + I3) my) p w
```

IS-LM AND *DA-OA* ANALYSIS

IS EQUATION

```
B = - M;
IS =
 FullSimplify[
  Solve[
   dy ==
    ReplaceAll[
     ReplaceAll[ReplaceAll[ReplaceAll[dc + dIn + dg + δ dk,
        {dc -> C1 (dy - dT - δ dk - (pi (p (dM + dB) - (M + B) dp) / p^2) + dpi (M + B) / p) + C2 (dr - dpi),
         dIn -> I1 dk + I2 dN + I3 (dr - dpi)}], {dk -> 0}], {dp / p → dw / w - Fnn dy / Fn^2}], {dN -> dy / Fn}], dr]]
```

$$\left\{\left\{dr \to dpi + \frac{-dg\, Fn + (C1\, dT + dy - C1\, dy)\, Fn - dy\, I2}{Fn\, (C2 + I3)}\right\}\right\}$$

```
IS = dr /. IS;
FullSimplify[D[IS, dy]]
```

$$\left\{\frac{Fn - C1\, Fn - I2}{C2\, Fn + Fn\, I3}\right\}$$

LM EQUATION

```
LM = FullSimplify[Solve[dy == ReplaceAll[((dM - M dp) / p^2 - mr dr) / my , {dp / p → dw / w - Fnn dy / Fn^2}], dr]]
```

$$\left\{\left\{dr \to \frac{dM - dp\, M - dy\, my\, p^2}{mr\, p^2}\right\}\right\}$$

```
LM = dr /. LM;
D[LM, dy]
```

$$\left\{-\frac{my}{mr}\right\}$$

DA EQUATION

```
B = - M;
DA = FullSimplify[Solve[IS == LM, dp]]
```

$$\left\{\left\{dp \to -\frac{mr\, p^2 \left(dpi + \frac{-dg\, Fn + C1\, (dT - dy)\, Fn + dy\, (Fn - I2)}{Fn\, (C2 + I3)} + \frac{-dM + dy\, my\, p^2}{mr\, p^2}\right)}{M}\right\}\right\}$$

```
DA = dp /. DA;
FullSimplify[D[DA, dy]]
```

$$\left\{\frac{((-1 + C1)\, Fn\, mr + I2\, mr - Fn\, (C2 + I3)\, my)\, p^2}{Fn\, (C2 + I3)\, M}\right\}$$

OA EQUATION

$$OA = dk \left(Fk + \frac{Fn\, Fnk\, Ns}{1 - Fnn\, Ns}\right)$$

$$dk \left(Fk + \frac{Fn\, Fnk\, Ns}{1 - Fnn\, Ns}\right)$$

```
D[OA, dp]
0
```

APÉNDICE I

Notación empleada en los modelos clásico y keynesiano.

Y = Nivel de ingreso (producción) de la economía.

K = Stock de capital de la economía.

N = Nivel de empleo de la economía.

P = Precio (o nivel de) del único bien producido en la economía.

w = Salario nominal de mercado o costo unitario del trabajo (igual al costo marginal del factor).

r = Rendimiento nominal de los bonos públicos.

π = Tasa esperada de incremento de los precios por parte del público o tasa real esperada del rendimiento del dinero, no necesariamente igual a la tasa efectiva de depreciación del mismo.

δ = Tasa instantánea de depreciación física del capital.

$r - \pi$ = Rendimiento esperado de los bonos públicos.

$r - \pi + \delta$ = Costo del capital de las empresas.

$-\dot{p}/p$ = Rendimiento real del dinero (supuesto que el rendimiento nominal del dinero nulo).

$r - \dot{p}/p$ = Rendimiento real de los bonos públicos.

C = Demanda de consumo de las economías domésticas.

I = Demanda de inversión de las empresas.

B = Valor nominal de los bonos públicos.

G = Gasto público en bienes, de uso inmediato no generando acumulación.

T = Impuestos reales, netos de transferencias, tipo Lump sum (de suma fija).

M = Cantidad nominal de dinero.

m_Y = Sensibilidad de la demanda real de dinero al ingreso de la economía.

m_r = Sensibilidad de la demanda real de dinero al rendimiento nominal de los bonos públicos.

N^S = Oferta laboral de la economía.

$F()$ = Tecnología de producción.

n = Número de empresas existentes.

APÉNDICE II
Notación empleada en Mathematica 4.0

Definición en el texto = definición en el lenguaje de Mathematica 4.0

$Y = y$

$K = K$

$N = N$

$P = p$

$w = w$

$r = r$

$\pi = pi$

$\delta = \delta$

$C = c$

$I = In$

$B = B$

$G = g$

$T = T$

$M = M$

$m_Y = my$

$m_r = mr$

$N^S = N^S$

$\partial N^S \big/ \partial \left(w/P \right) = Ns$

$w/P = wp$

$F() = F$

$F_N = Fn$

$F_K = Fk$

$F_{NN} = Fnn$

$F_{KK} = Fkk$

$F_{NK} = F_{KN} = Fnk = Fkn$

$I_1 = I1$

$I_2 = I2$

$I_3 = I3$

$I_4 = I4$

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